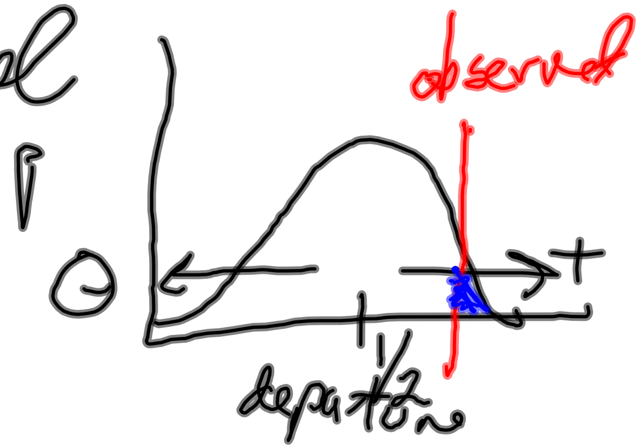
Bayesian Analysis

Non-Bayes: hypothesis testing

null model ≡ "hypothesis"

Q: what is robability of data (actual) 'if' it arose under the null model

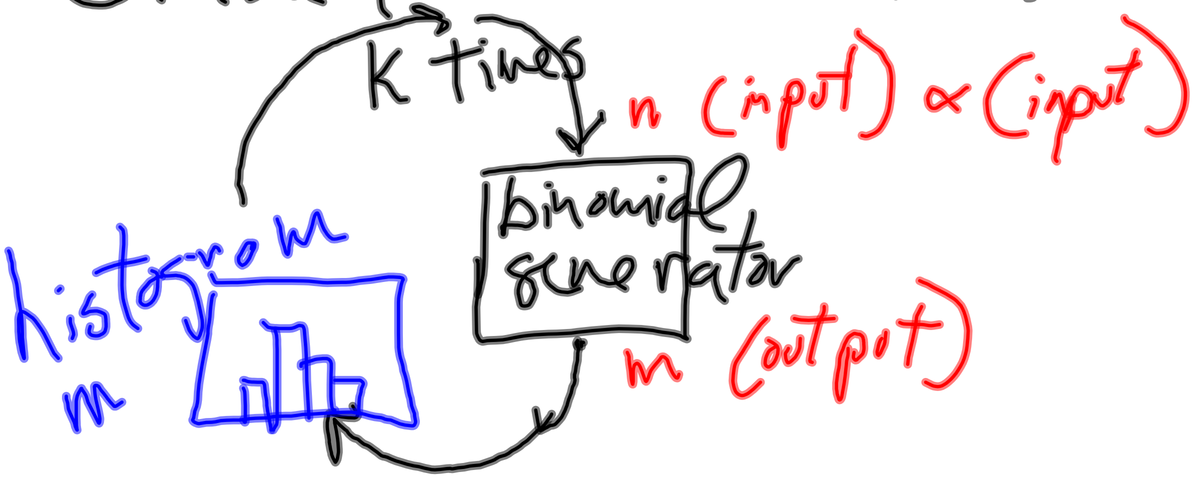


ratio $\frac{\text{heads}}{\Sigma\,\text{Total}}$

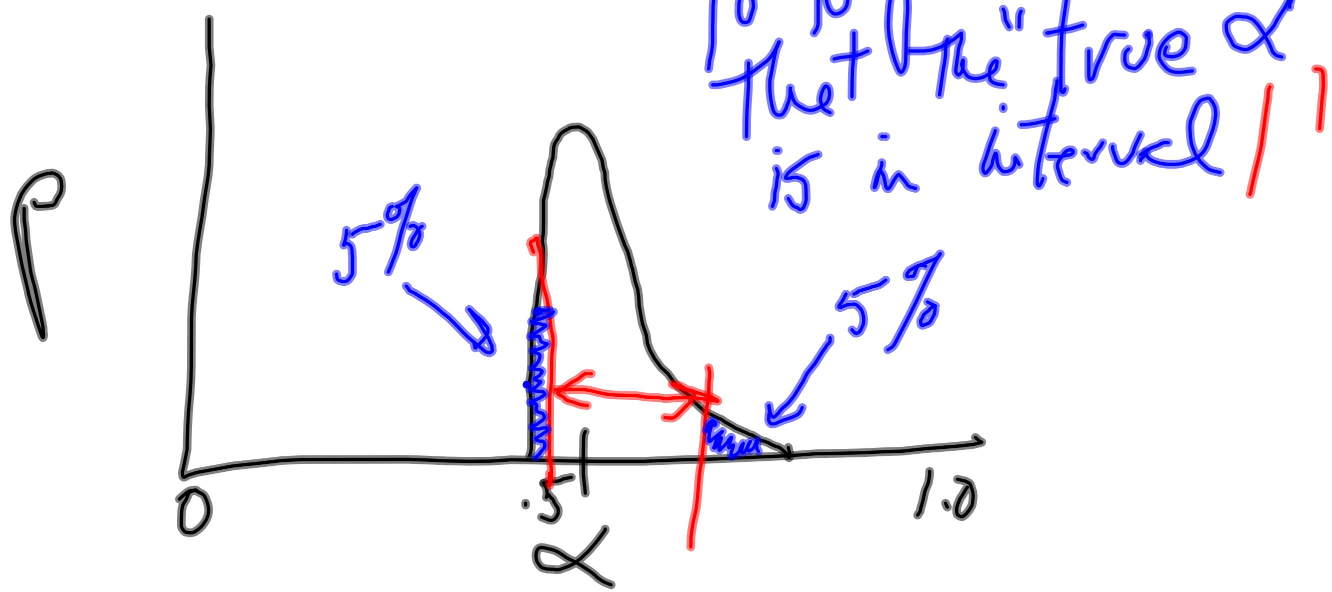
Binomial distribution

$$p(m \mid n, \alpha) = \alpha^{m}(1-\alpha)^{n-m}\frac{n!}{m!\,(n-m)!}$$

Simulation alternative:



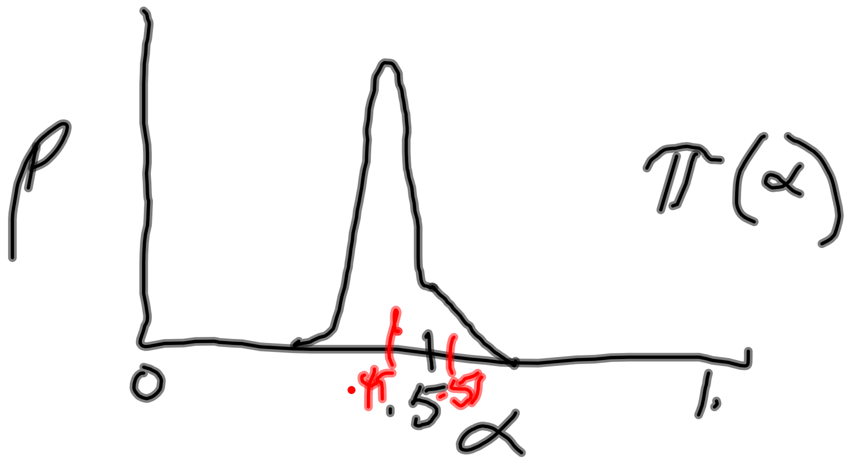
Bayesian inference:
Estimate parameter



Probability that next spin is heads when our knowlege about this coin is a probability distribution for α'.

α is probability of heads from spinning

Coins in general



I. p of heads on next toss from coin drawn from the bucket

$$E(\alpha) = \int_0^1 \alpha \, \pi(\alpha) \, d\alpha$$

$$\sum_i \alpha_i \frac{n_i}{n_\Sigma} = \bar{\alpha}$$

Probability of 2 heads in next 2 tosses:

$$\left(E(\alpha)\right)^2 \overset{?}{=} \int_0^1 \alpha^2 \pi(\alpha)\, d\alpha = E(\alpha^2)$$

no

$$Var(x) = Av\left(x - \bar{x}\right)^2 = Av\left(x^2 - 2x\bar{x} + \bar{x}^2\right)$$

$$Av(x^2) - A(2x\bar{x}) + A(\bar{x}^2)$$

$$= Av(x^2) - 2\bar{x}^2 + \bar{x}^2 = Av(x^2) - \bar{x}^2$$

$$Av(x^2) = Var(x) + \bar{x}^2$$

random process

$$\text{governing parameters} \;\underset{\text{statistics}}{\overset{\text{probability}}{\rightleftarrows}}\; \text{outcome}$$

Game II

Pick a coin from bucket
spin it 5 times, observe
results, place bet on
next spin

"$\mathcal{L}$"ikelihood function

$$\mathcal{L}_m(\alpha) \equiv p(m|\alpha)$$

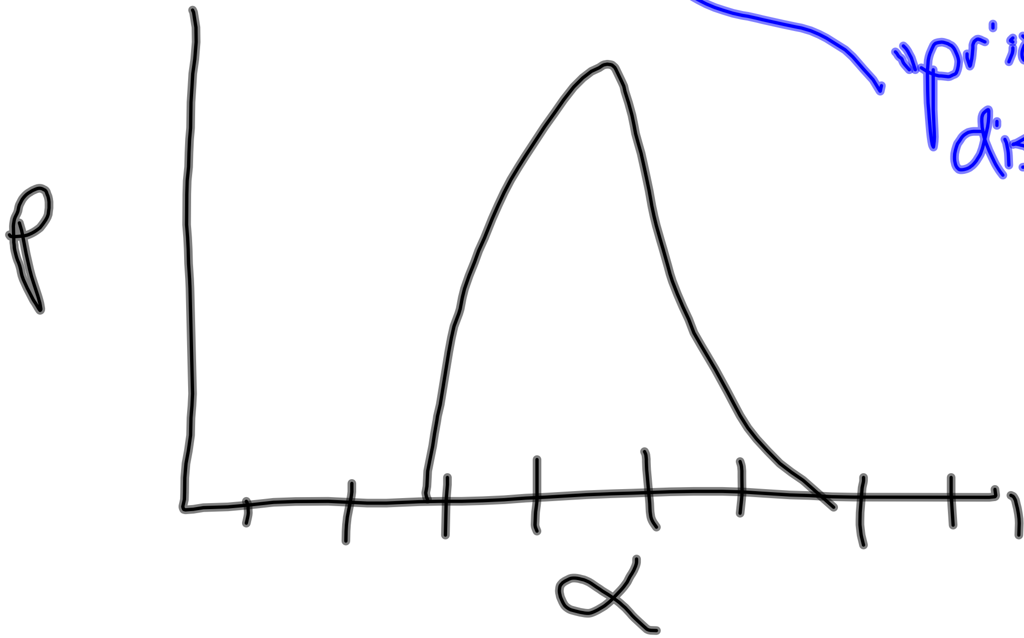
$$p(\alpha|m) = \frac{\pi(\alpha)\,\mathcal{L}_m(\alpha)}{\int_0^1 \pi(\alpha)\,\mathcal{L}_m(\alpha)\,d\alpha}$$

$$p(\alpha \mid m, n) = \frac{\pi(\alpha)\, p(m \mid n, \alpha)}{\int \text{num}\, d\alpha}$$

"$\mathcal{L}$"

"posterior distribution"

"prior distribution"



$$p(\alpha|x) \propto \pi(\alpha)\, p(x|\alpha)$$

$$p(\alpha|x,y) \propto \pi(\alpha)\, p(x,y|\alpha)$$

$$p(\alpha|x,y) \propto p(\alpha|x)\, p(y|\alpha)$$

$$\propto \underbrace{\underbrace{\pi(\alpha)}\,\underbrace{p(x|\alpha)}}\,\underbrace{p(y|\alpha)}$$