PATH AVERAGE IN A TRAJECTORY

Given N_0 and N_T

Realized Factor of increase over the T time interval is $\frac{N_T}{N_0}$

This equivalent to growing at a constant $\hat{\lambda}$ (per year) for T years

$\hat{\lambda} = ?$

$$\hat{\lambda}^{T} = \frac{N_T}{N_0}$$

$$\hat{\lambda} = \left(\frac{N_T}{N_0}\right)^{\frac{1}{T}}$$

$$\ln \hat{\lambda} = \frac{1}{T} \ln \left(\frac{N_T}{N_0}\right)$$

$$= \frac{1}{T}\left(\ln N_T - \ln N_0\right)$$

How does this relate to the actual realized sequence of λ's (if they weren't constant)

$$N_T = N_0 \prod_{j=1}^{T} \lambda_j$$

$$\hat{\lambda}^T = \frac{N_T}{N_0} = \prod_{j=1}^{T} \lambda_j$$

$$T \ln \hat{\lambda} = \sum_{j=1}^{T} \ln \lambda_j$$

$$\ln \hat{\lambda} = \frac{1}{T} \sum_{j=1}^{T} \ln \lambda_j$$

$$= \text{mean of } \ln \lambda\text{'s}$$

For 1 time step

ensemble $\bar{\lambda}$ and σ^2_λ

$$N_1 = N_0 \lambda$$

$$\frac{N_1}{N_0} = \lambda$$

$$E\left(\frac{N_1}{N_0}\right) = \bar{\lambda}$$

What about distribution of $\ln(N_1)$?

$$\ln(N_1) - \ln(N_0)$$

$$= \ln\left(\frac{N_1}{N_0}\right)$$

For any given N_1

$$\ln\left(\frac{N_1}{N_0}\right) = \ln \lambda$$

So the distribution
of $\ln\left(\frac{N_1}{N_0}\right)$
which is the distribution
of $\ln(N_1) - \ln N_0$
is the distribution of
$\ln\lambda$ so its mean gives?
$= \ln$ of geometric mean of λ

Recall geometric mean of x is $\left(\prod^{k} x_j\right)^{1/k}$

$$\ln\left(\text{geometric mean}\right) = \frac{\sum \ln x}{k}$$

Call $\hat{\lambda}$ the geometric mean

$$\ln \hat{\lambda} = \text{mean}\left(\ln N_1\right) - \ln N_0$$

PVAX output

$$\hat{\lambda} = \frac{e^{\text{mean}\left(\ln N_1\right)}}{N_0}$$

(ensemble)